

13/12/2015
 [9] راجه هندسيه
 AMP
 M.T
 [9]

IR. S. P

$$x(P(x)) \Big|_{x=0} = \infty$$

$$x^2(Q(x)) \Big|_{x=0} = \infty$$

ايضا لها حل .

R. S. P

$$x(P(x)) \Big|_{x=0} \neq \infty$$

$$x^2(Q(x)) \Big|_{x=0} \neq \infty$$

$$\Rightarrow y = \sum a_n x^{n+\lambda} \rightarrow y_{G.S} = c_1 y_1 + c_2 y_2$$

$$a\lambda^2 + b\lambda + c = 0$$

$$\lambda_1 - \lambda_2 \neq \text{integer}$$

$$y_1 = x^{\lambda_1} [a_0 + a_1 x + \dots]$$

$$y_2 = x^{\lambda_2} [a_0 + a_1 x + \dots]$$

$$\lambda_1 = \lambda_2 = \lambda$$

$$y_1 = x^{\lambda} [a_0 + a_1 x + \dots]$$

$$y_2 = \frac{\partial y(x, \lambda)}{\partial \lambda} \Big|_{\lambda}$$

$$\lambda_1 - \lambda_2 = \text{integer}$$

$$y_1 = x^{\lambda_1} [a_0 + a_1 x + \dots]$$

$$a_n \neq \infty$$

$$a_n = \infty$$

نكتفي بطرف واحد

$$y_2 = \frac{\partial (\lambda - \lambda_2) y_1}{\partial \lambda} \Big|_{\lambda = \lambda_2}$$

[b] Ex 2

$$x^2 y'' + x y' + \left(x^2 - \frac{4}{9}\right) y = 0$$

$$y'' + \frac{1}{x} y' + \frac{\left(x^2 - \frac{4}{9}\right)}{x^2} y = 0$$

$$\lim_{x \rightarrow 0} P(x) = \frac{1}{x} \rightarrow \infty$$

$$\lim_{x \rightarrow 0} Q(x) = \frac{x^2 - \frac{4}{9}}{x^2} = \infty$$

$$x \text{ في } P(x) \text{ تقرب } \rightarrow x + \frac{1}{x} = 1$$

$$x^2 \text{ في } Q(x) \text{ تقرب } \rightarrow x^2 + \frac{x^2 - \frac{4}{9}}{x^2} = -\frac{4}{9} \quad \left. \vphantom{x^2 + \frac{x^2 - \frac{4}{9}}{x^2}} \right\} \underline{\text{R.S.P}}$$

$$\infty \quad y = \sum a_n x^{n+\lambda}$$

$$y' = \sum_{n=0}^{\infty} (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\begin{aligned} & \sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} + \sum (n+\lambda) a_n x^{n+\lambda} \\ & + \sum a_n x^{n+\lambda+2} - \frac{4}{9} \sum a_n x^{n+\lambda} = 0 \end{aligned}$$

$n=0$ المعادلة الأولى مفقودة

المتشابهة الأولى عدد

$$C_0 \text{ f } x^\lambda \rightarrow \lambda + (\lambda-1) a_0 + \lambda a_0 - \frac{4}{9} a_0 = 0$$

$$a_0 \left(\cancel{\lambda^2 - \lambda} + \lambda - \frac{4}{9} \right) = 0$$

$$\text{لذا } a_0 = 0$$

وهذا مفقود

$$\left. \vphantom{\lambda^2 - \frac{4}{9} = 0} \right\} \text{لذا } \lambda^2 - \frac{4}{9} = 0 \rightarrow \lambda = \pm \frac{2}{3}$$

$$\lambda_1 - \lambda_2 = \text{integer} \implies \text{so } \lambda_1 \in \mathbb{Z}$$

$$\text{Co. of } x^{\lambda+1} \longrightarrow (\lambda+1)(\lambda)a_1 + (\lambda+1)a_1 - \frac{4}{9}a_1 = 0$$

$$a_1(\lambda^2 + \lambda + \lambda + 1 - \frac{4}{9}) = 0$$

$$V_1 \quad \left(\begin{array}{c} a = a_1 V_1 \\ (9) \end{array} \right)$$

$$\boxed{a_1 = 0}$$

$$\text{Co. of } x^{n+\lambda} \longrightarrow (n+\lambda)(n+\lambda-1)a_n + (n+\lambda)a_n + a_{n-2} - \frac{4}{9}a_n = 0$$

$$a_n = \frac{-(a_{n-2})}{(n+\lambda)(n+\lambda-1) + (n+\lambda) - \frac{4}{9}} = \frac{-(a_{n-2})}{(n+\lambda)^2 - \frac{4}{9}}$$

$$\text{at } n=2 \quad a_2 = \frac{-a_0}{(\lambda+2)^2 - \frac{4}{9}}$$

$$\text{so } y_1 = x^{\frac{2}{3}} \left[a_0 - \frac{a_0}{(\frac{2}{3}+2)^2 - \frac{4}{9}} x^2 + \dots \right]$$

$$y_2 = x^{-\frac{2}{3}} \left[a_0 - \frac{a_0}{(\frac{-2}{3}+2)^2 - \frac{4}{9}} x^2 + \dots \right]$$

$$y_{G.S} = C_1 y_1 + C_2 y_2$$

d

$$x^2 y'' - x y' + (1+x)y = 0$$

$$P(x) = \frac{-1}{x} = \infty \quad * \quad x = -1 \neq \infty$$

$$Q(x) = \frac{1+x}{x^2} = \infty \quad * \quad x^2 = 1 \neq \infty$$

R.s.p

$$y = \sum a_n x^{n+\lambda}$$

$$y' = \sum (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = \sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} - \sum (n+\lambda) a_n x^{n+\lambda} + \sum a_n x^{n+\lambda} + a_n x^{n+\lambda+1} = 0$$

$$n = 0 \text{ نلاحظ } \longrightarrow x^\lambda \longrightarrow \text{أول حد}$$

$$\text{Co. of } x^\lambda \longrightarrow \lambda * (\lambda-1) a_0 - \lambda a_0 + a_0 = 0$$

$$a_0 (\lambda^2 - \lambda - \lambda + 1) = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$\lambda_1 = \lambda_2 = \lambda = 1 \longrightarrow$$

في الحالة الثانية

$$\text{Co. of } x^{\lambda+1} \longrightarrow (\lambda+1) * \lambda a_1 - (\lambda+1) a_1 + a_1 + a_0 = 0$$

$$a_1 = \frac{a_0}{(\lambda^2 + \lambda - \lambda - 1 + 1)} = \frac{a_0}{\lambda^2}$$

Co. of $x^{n+\lambda}$

e

$$\rightarrow (n+\lambda)(n+\lambda-1)a_n - (n+\lambda)a_n + a_n + a_{n-1} = 0$$

$$a_n = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-1) - (n+\lambda) + 1}$$

$$a_n = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-1-1) + 1} = \frac{-(a_{n-1})}{(n+\lambda)(n+\lambda-2) + 1}$$

$$a_2 = \frac{-a_1}{(\lambda+2)(\lambda) + 1}$$

$$y_1 = x \left[a_0 - \frac{a_0}{1} x - \frac{a_1}{4} x^2 + \dots \right]$$

$$y(x, \lambda) = x^\lambda \left[a_0 - \frac{a_0}{\lambda^2} x + \frac{a_0}{\lambda^3(\lambda+2) + \lambda^2} x^2 + \dots \right]$$

$$y_2 = \frac{\partial y(x, \lambda)}{\partial \lambda} = x^\lambda \left[+2a_0 x \lambda^{-3} + \dots \right] +$$

$$\underbrace{x^\lambda}_{\text{term}} + \underbrace{1}_{\text{constant}} + \underbrace{\ln \lambda}_{\text{integral}} \left[a_0 + \frac{a_0}{\lambda} x - \frac{a_1}{4} x^2 + \dots \right]$$

F Ex. $x^2 y'' + (x^2 - 2x)y' + 2y = 0$

$$P(x) = \frac{x^2 - 2x}{x^2} = \infty$$

$$Q(x) = \frac{2}{x^2} = \infty$$

$$\left. \begin{array}{l} \rightarrow * x \rightarrow \frac{x^2 - 2x}{x} = -2 \neq \infty \\ \rightarrow * x^2 \rightarrow 2 = \infty \end{array} \right\} \text{R.s.p.}$$

$$y = \sum a_n x^{n+\lambda}$$

$$y' = (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} + \sum (n+\lambda) a_n x^{n+\lambda+1} - 2 \sum (n+\lambda) a_n x^{n+\lambda} + 2 \sum a_n x^{n+\lambda} = 0$$

$$\text{co. of } x^\lambda \rightarrow \lambda(\lambda-1)a_0 - 2\lambda a_0 + 2a_0 = 0$$

$$a_0(\lambda^2 - \lambda - 2\lambda + 2) = 0$$

$$\lambda = 1, 2 = \text{في الحالة الثالثة} \rightarrow \text{عدد صحيح}$$

$$\text{co. of } x^{\lambda+1} \rightarrow (\lambda+1)(\lambda) a_1 + \lambda a_0 - 2(\lambda+1) a_1 + 2a_1 = 0$$

$$a_1 = \frac{-\lambda a_0}{(\lambda^2 + \lambda - 2\lambda - 2 + 2)} = \frac{-a_0}{\lambda+1}$$

Co-ef $x^{n+\lambda}$

3

$$(n+\lambda)(n+\lambda-1)a_n + (n+\lambda-1)a_{n-1} - 2(n+\lambda)a_n + 2a_{n-1}$$

$$a_n = \frac{-(n+\lambda-1)a_{n-1}}{(n+\lambda)(n+\lambda-1) - 2(n+\lambda) + 2} = \frac{-(n+\lambda-1)a_{n-1}}{(n+\lambda)(n+\lambda-3) + 2}$$

$$a_2 = \frac{-(\lambda+1)a_1}{(\lambda+2)(\lambda-1) + 2}$$

نكمل في الحالة الثالثة